ON A DIAGONAL CRACK NUMERICAL MODEL OF RC BEAM WITH NO SHEAR REINFORCEMENT

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1. Introduction

The prediction of the ultimate shear strength of reinforced concrete (RC) beams is very important especially when this value is used in the practical design. An unconservative value of the shear capacity may lead to an unexpected and at early stage brittle failure of the structural RC beam so, a great deal of research efforts have been put recently on the proper explanation, modeling and simulation of the shear crack phenomena [1], [5]. Although the current design codes for shear in RC beams rely almost entirely on the test results, a number of simplified methods are given in [1], based on theory of plasticity and the relevant effectiveness factors.

It is fair to state that the mechanism of the brittle type diagonal tensile failure of RC beams with no shear reinforcement (stirrups) is complex and not fully understood (see Fig. 1)

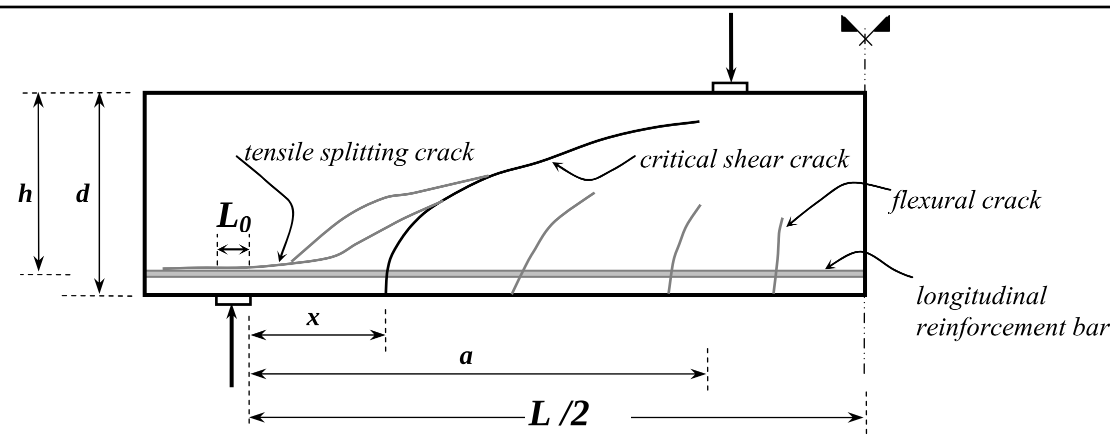

Fig. 1. *Shear – tension failure mode of RC beam*

This behaviour is common to slender beams – these are beams with ratio a/h (shear span to effective height) between 2 and 4. There are of course few other important factors such as reinforcement ratio and others. Diagonal shear failure starts with the development of few vertical flexural cracks at the midspan, followed by a destruction of the bond between the reinforcement steel and concrete at the zone of the support (see Fig. 1.). Thereafter, without ample warning of a failure, one major diagonal crack develops (we call it critical crack) at about (1.5-2)d distance from the support. Depending on some material or geometric parameters of the beam, the critical crack may extend to the top of the compression fibers and then stabilize. That happens at comparatively small deflection and is considered to be the shear capacity of the RC beam under consideration.

2. Comments on some results developed in work [2]

Considerable efforts have been devoted in recent years of developing numerical methods and models to simulate the real behaviour of quassi-brittle materials, such as mortar, concrete and bricks used in civil engineering structures. Traditionally, the numerical models are based on the finite (FEM) or boundary (BEM) elements and are classified into two groups: “smeared” crack approach and “discrete” crack approach. In the smeared crack models the fracture or crack is represented in a smeared over a finite area manner. Without going into detail we shall mention that the present research team has developed an extensive numerical research on RC shear beams using smeared approach and ANSYS software program [2]. A 3D brick concrete element was used and reinforcement bars were modeled again by smearing out the steel over the concrete elements. For comparison the RC beam tested and numerically examined by Hibino at al. (see Fig. 2.) was used.

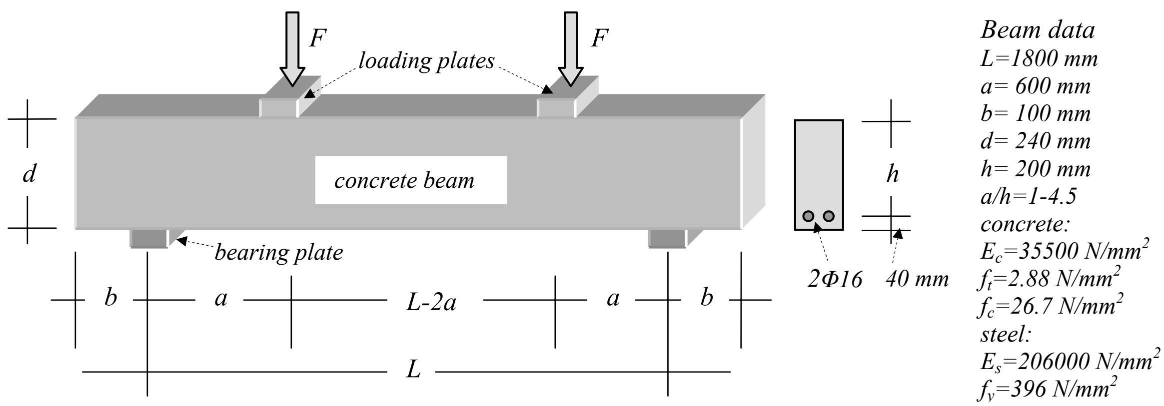

Fig. 2. *Geometric and material data for the reinforced concrete beam with no shear reinforcement analyzed in paper [4]. The shear span ratio in this case a/h is changing*

The conclusion drawn from the research published in [2} are as follows:

- The general conclusion is that using 3D ANSYS modeling we are able to properly simulate the nonlinear behaviour of R/C beams without shear reinforcement having a moderate shear span size (*a/h=3* for the beam under consideration);
- ANSYS 3D concrete element is very good concerning the flexural and shear crack development but poor concerning the crushing state. However this deficiency could be easier removed by employing a certain multilinear plasticity options available in ANSYS;
- The particular concrete finite element does not consider one of the most important fracture mechanics parameter – that is the fracture energy G_F. That means that in the case of concrete beam with no reinforcement we will be not able to get a proper solution;
- By using ANSYS smeared approach for beams with moderate shear span *we are not able to reproduce satisfactory the softening due to big sliding emerging at the critical shear crack*. That is likely to be more realistically achieved by 2D discrete crack approach;
- The results and the parametric study (not given in the paper [2]) suggest that we need some correction factors to adjust the values of material parameters available from the experiment and convert them to effective parameters related to the particular modeling;
- Therefore, much more research is needed in order to: (1) develop a similar simulation for R/C deeper beams; (2) suggest reliable methods for adjusting the experimental material data to effective parameter data suitable to particular finite element models.

The above conclusion immediately suggested that a new 2D "discrete" crack model should be developed in order to handle to big amount of sliding which is attributed to the development of the critical crack. The ANSYS program and its nonlinear options and capabilities are employed again to achieve this aim.

3. The present modeling

An important feature of the present 2D modeling is that the flexural (mainly vertical), shear/flexural and the critical shear crack are prescribed (see Fig. 3. a, b), which means that their position and length must be determined in advance using a certain method. In this research we employ the linear fracture mechanics approach developed in a previous research [3] by the present authors. A 2D isoparametric ANSYS element with 16 DOF is used to represent the concrete continuum and a linear truss element with plasticity options to model the reinforcement bars. Omitting details we start with the two equations of the linear elastic fracture mechanics needed to determine the crack path [3]:

$$K_I \cos^2\frac{\theta_c}{2}\sin\frac{\theta_c}{2} + K_{II}\cos\frac{\theta_c}{2}\left(\cos^2\frac{\theta_c}{2} - 2\sin^2\frac{\theta_c}{2}\right) = 0\,,$$
$$K_I \cos^3\frac{\theta_c}{2} - 3K_{II}\cos^2\frac{\theta_c}{2}\sin\frac{\theta_c}{2} = K_{Ic}\,, \qquad (1)$$

where K_I and K_{II} are the stress intensity factors for mode I and mode II respectively, K_{Ic} is the critical value of K_I considered to be a constant taken from the experiment, and θ_c is the angle of crack direction in which the tensile stress is principal and maximum.

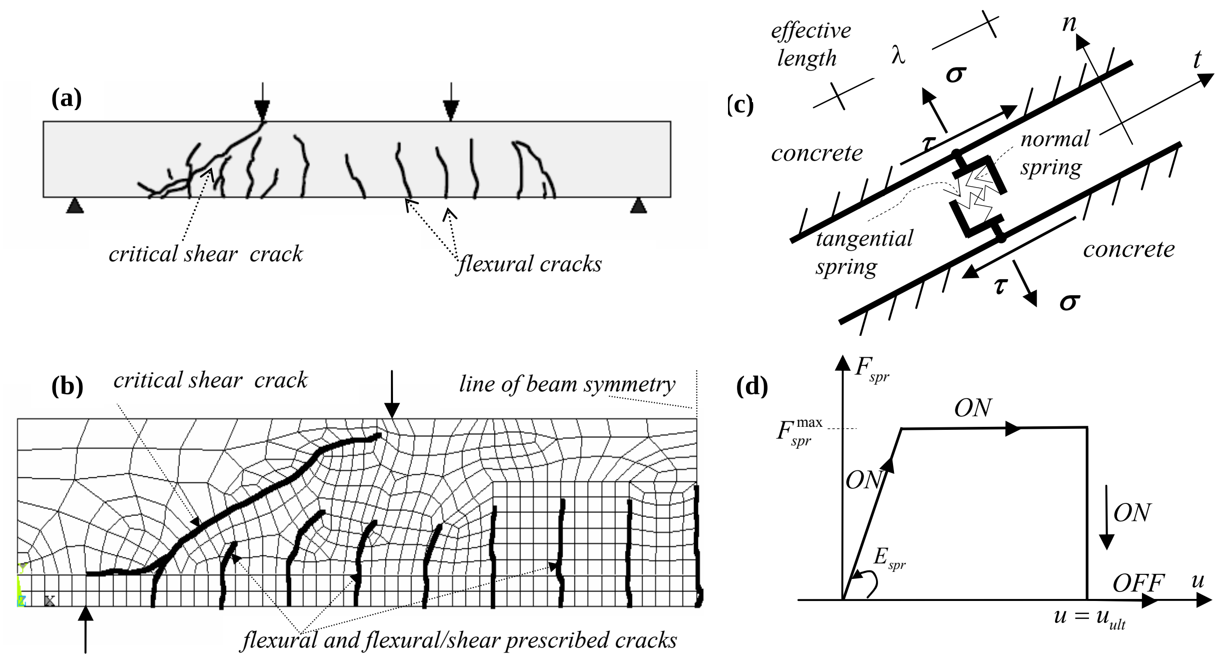

***Fig. 3. (a)** Crack development picture taken from the test [4]; **(b)** Finite element mesh and prescribed flexural and shear cracks; **(c)** Formulation of the translational springs; **(d)** Accepted constitutive law of springs and behaviour of COMBIN37 nonlinear "control" spring element*

In Fig. 3. b., the half part of the beam is given with the finite element mesh and the prescribed flexural and shear cracks obtained by means of the above method. The next step in the modeling is to develop the normal and tangential nonlinear springs (see Fig. 3. c) and put them at the contact interfaces of the cracks. Their job is to simulate the complex nonlinear processes which are happening in the cracks as the loading process is progressing.

The formulation of the constitutive law of the normal springs depends on the fracture energy parameter for mode I and since it is not known from the experiment, we calculate it from the well known formula:

$$G_F^I = 0.03(f_c/10)^{0.7} \quad , \qquad (2)$$

where f_c is the uniaxial compression strength of the concrete, so the final value is $G_F^I = 0.03(26.7/10)^{0.7} = 0.0596$ (N/mm), which leads to critical value of normal displacement (see Fig. 3. d) of approximately 0.02 mm, used in the present numerical simulation. A special "control" nonlinear ANSYS spring element is chosen being able to perform some "controlling" functions during the nonlinear simulation process. The particular spring is turning ON or OFF (alive or dead) depending on defined in advance parameter which in this case is the crack opening displacement. For the tangential springs a value of 0.5 mm was chosen for the ultimate value of tangential displacement according to some experimental evidences. The elastic stiffness of the springs was chosen by means of the principle "maximum value which does not cause problems in convergency", so the values are as follows: $E_n = 5.3310^6\ N/mm$ for normal and $E_t = 2.2310^6\ N/mm$ for tangential springs. The calculation of the yielding normal spring force $F_n^{\max}$ was done by the well known approach involving the effective length λ (in our case $\lambda = 20\,mm$): $F_n^{\max} = \lambda b f_t = 20.150.2,88 = 8640\,N$. The value of the yielding tangential spring force was a matter of some theoretical and practical difficulties. It is obvious that this value is variable depending on the current value of the normal force. However ANSYS is not able to handle such a relationship. After many numerical experiments it was decided to calculate a maximum value based on the formula given in [4]: $\tau_{\max} = 014 f_c - tg\phi f_t = 0,14.26,7 - tg(37).2,88 = 1,56\,N/mm^2$, which led to value of $F_n^{\max} = 4680\,N$ for the tangential force. It is worth mentioning that the usual bilinear plasticity option for the steel and Drucker-Prager option for the concrete in compression were used in the numerical solution.

4. Numerical results

The numerical results and comparisons presented in this section are completely based on the experimental and numerical results given in reference [4]. The material and geometric data can be depicted from Fig. 2. Three solution are presented in [4] for different values of ratio a/h, namely a/h=2,3 and 3,6 (all other data is identical). In the present numerical simulation we emphasize on the following important research points:

- To get and compare (where possible) the value of the ultimate (critical, maximum) external force.
- To obtain and compare the full range relationship between the external force and middle span deflection;
- To get a proper picture of the type of the failure – pure bending, shear/tension, mixed etc..;

Number of solutions were performed for different values of a/h=2.05, 2.57, 3, 3.61, 4.13 – notice that for only 3 values: 2, 3 and 3.6 comparison with [4] can be made. In Fig. 4 the relative beam strength, represented as a ratio of the ultimate (real) and flexural moments, versus a/h ration is presented.

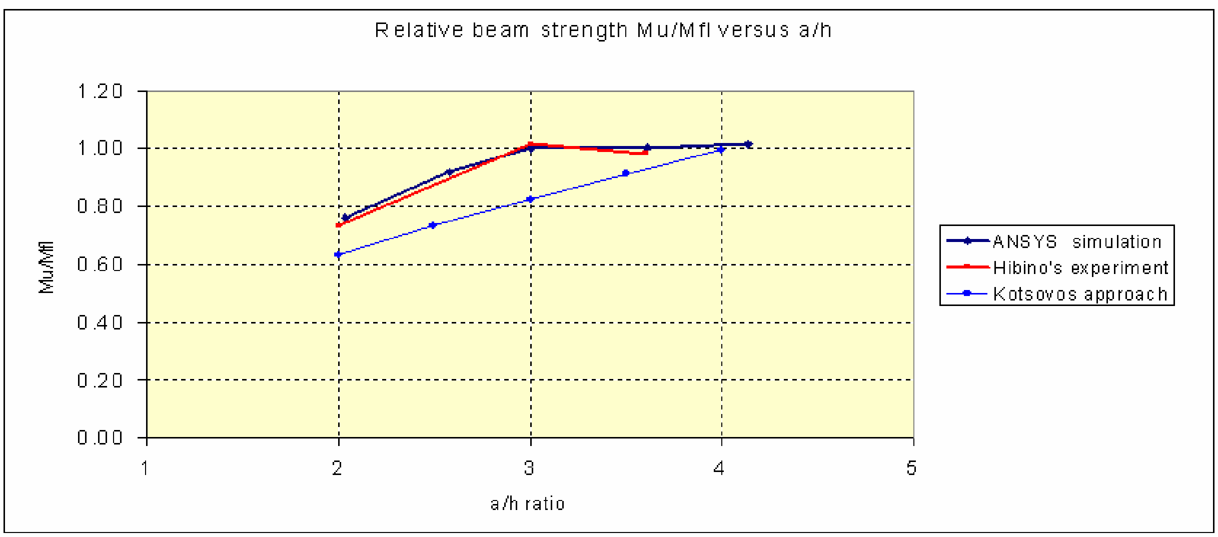

Fig. 4. *Relative beam strength Mu/M_{fl} versus a/d ratio. Comparison between present simulation, test results from [4] and the simple solution of Kotsovos [5]*

The flexural moments are considered constant for values of a/h > 3.5. It is also usually presumed that the reinforcement is yielding, so we have development of a vertical crack reaching the compression upper zone. The type of failure is bending and ductile with no development of a huge critical shear crack. As seen from Fig.4 the value of a/h=3 is the boundary between the bending (ductile) and shear/tension (brittle in some cases) type of the failure. That is the point where there is much pronounced interaction between beam and arch actions. The type of failure could be brittle and relative beam strength M_u/M_{fl} is decreasing. A critical shear crack is usually developing after this value of a/d parameter, at least that was the case we have observed in the present simulation. The failure was of shear/tensile manner although the reinforcement steel was in a linear elastic phase. However we must mention an interesting detail. Firstly we made numerical simulation ignoring the horizontal part of the critical shear crack. As a result the tangential springs put into the critical shear crack did not manage to "escape" from their linear branch and consequently develop into a nonlinear one. As the horizontal part of the crack was available the tangential springs began working properly as expected.

From the graphics of Fig. 4 it is evident that in overall not bad coincidence exists between present numerical and experimental results. On the other hand the results obtained with a simple design formula given in the book of Kotsovos [5] are much far from the experimental. In Fig. 5. external force – middle beam deflection relationship is drawn for the case of ratio a/h=3. The three graphics presented are as follows: (1) present simulation; (2) experimental graphic of Hibino et al.; (3) numerical solution graphic of Hibino et al.

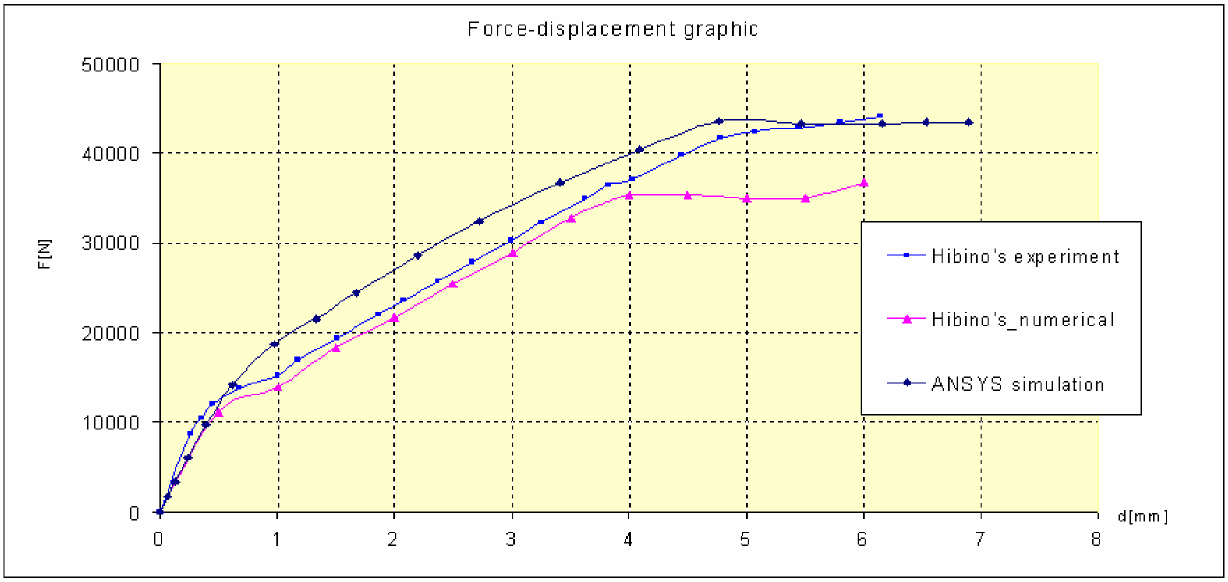

Fig. 5. *Force – middle beam displacement relationship. Comparison between present solution and Hibino's (a/h=3) experimental and numerical data [4]*

Although the fitting between experimental and present numerical graphics is not so good, the value of the ultimate load is very closed to the one obtained from the experiment. It is quite clear from the figure that the flexural cracks response and development is present and that the variation of the beam stiffness is reproduced well.

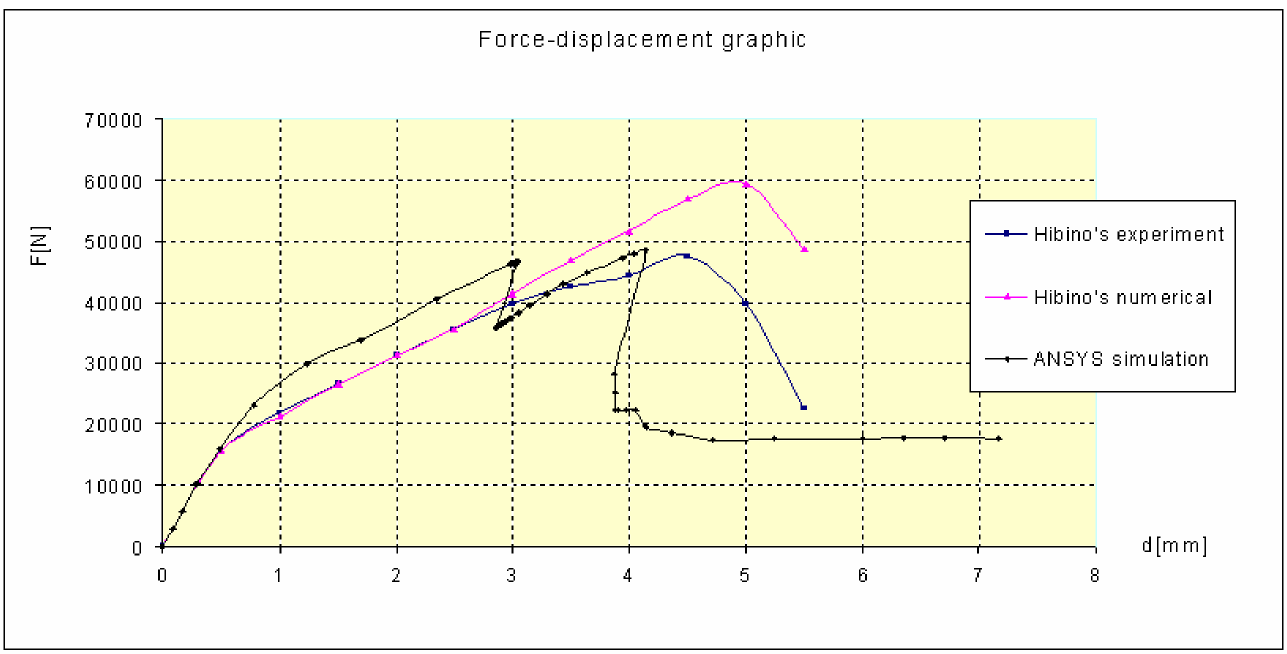

Fig. 6. *Force – middle beam displacement relationship. Comparison between present solution and Hibino's (a/h=2) experimental and numerical data [4]*

At about 3.5-5 mm deflection a plasticity processes in the compression zone is developing and after the steel began to yield the deflection is growing with no further loading.

It is interesting to mention that this case is just at the boundary between bending and shear failure type. The normal springs put at the critical shear crack are exhausted but not the tangential springs, so tangential sliding does not develop for this case. That is happening for values of a/h less then 3 as it can be observed in Fig.6, where an interesting force – deflection curve is plotted for ratio a/h=2. The ANSYS simulation graphic shows typical shear/tension failure with two significant drops of the force. The first one is due to massive opening of the normal springs at the location of the critical shear crack. There is stiffening part of the curve followed by a continuous softening of the tangential springs at about 4 mm deflection. That is happening at about 1 mm earlier compared to the experiment and is accompanied with big relative tangential displacements (sliding). Even at that final stage the reinforcement steel is working in a linear elastic regime. Although the fitting with the experiment is not good the ultimate value of the external force is about the same as in experiment.

5. Discussion

This work is part of a research project devoted to development of new numerical models for nonlinear analysis of structures made from quasibrittle materials like concrete using fracture mechanics approach. Based on the numerical results we draw the following main conclusions from this numerical research:

- The present variant of the modeling is based on prescribed in advance discrete cracks and nonlinear translational springs put at the crack interface. The discrete crack path is developed by the means of the linear fracture mechanics and the results have shown a good fitting with the experimentally observed;
- In general, this 2D numerical simulation is capable to successfully simulate the real behaviour of RC beams for different values of a/d ratio;
- The type of failure, the development of the critical shear crack as well as the ultimate external load are captured well, but not the fitting with the experimental load-deflection curve within the full range of displacements;
- The plasticity in reinforcement and concrete in the compression zone are reproduced with sufficient accuracy including the final failure phase of the simulation;
- The main drawback of the suggested model is the way the constitutive data for the tangential springs is extracted. The existing and important relationship between the normal and tangential relative displacements is not modeled, therefore an important phenomena such as dilatation is not present in the model;
- A new better constitutive mode, independent from those available in ANSYS, should be developed. Probably the new model should be based on the damage or softening plasticity theory. The relationship between normal and tangential stresses will be related through a "failure surface", so it is natural to get the proper displacements relationship. Therefore, the dilatancy phenomena may be handled and controlled properly.

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