Numerical analysis of anchored diaphragm walls
(Relationship between FEM and SRM)

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ABSTRACT

Abstract text

An approach to surmounting some difficulties related to the application of the Subgrade Reaction Method (SRM) by calculating one-level anchored (supported) retaining walls is presented. The numerical analysis is performed using the specialized geotechnical program PLAXIS 7.2 and the universal program for static analysis SAP2000. The main problems arising in determining the type, size and distribution of load and support as well as possible variants by anchor modeling are analyzed.

Keywords: retaining walls, Subgrade reaction method, anchors

1 INTRODUCTION

The excavation of foundation pits along the boundary of existing buildings and/or built-up street infrastructure is related to executing the respective support structure. On the territory of Bulgaria these structures are mostly pile or diaphragm walls supported (anchored) at one or more levels. A more detailed analysis has shown that 2/3 of the supported foundation pits in Bulgaria are executed as structures anchored or supported at one level. In special literature as well as technical standards there is a wide variety of methods for designing retaining walls . This versatility affects the results of the calculations which vary within a broad range for the various methods (v.Wolffersdorff 1994 & 1996). The complexity of the problem can be explained by the fact that this type of structure is not only supported in a particularly complicated and non-homogeneous medium (soil) but also the load acting on it is formed by that very medium.

In designing one-level anchored retaining walls in Bulgaria, despite the fast development of specialized software products over the last few years, the most frequently used method in construction practice is the Subgrade Reaction Method (SRM) based on Winkler's hypothesis. The idea of modeling the interaction 'soil-structure' by using a proportionality coefficient considering the relationship between stresses and deformations is a simple idealization in modeling the actual operation of the structure. The advantage of this method lies in its relative simplicity and clarity.

The work with modern software packages accounting for the physical non-linearity of soils which depends on the change in their stress state; their action as a multi-phase system; the inclusion of an elastic and plastic component in determining the deformations; the design of constitutive models complying maximally to the actual behavior of the soil as a medium; all these factors improve considerably the accuracy in the overall modeling of engineering structures in soil.

The aim of the numerical study conducted and presented here is to search recommendations for designing one-level anchored retaining walls by using an arbitrary statistical analysis program through the realization of multiple parallel solutions applying SRM and PLAXIS for a broad range of possible input parameter changes. The main problems in applying the analysis are as follows:

- Determination of the distribution and size of active earth load;
- Determination of the distribution of Winkler's proportionality coefficient in depth and its accurate specification

according to the experimental data obtained on the particular soil mass;

- Modeling the anchor support.

The SRM consists in substituting the passive earth load by modeling the interaction between subgrade and structure with a series of untied spring supports of constant or varying stiffness in depth according to certain laws of stiffening in depth (Vogt 1984, Besler 1998, etc.). Besides determining the type and distribution of load and support pattern, the yieldability of the anchor support has a considerable effect on the accuracy of the solution and correctness of the results obtained. The results of the preliminary calculations performed (Ilov & Totsev 2005) show explicitly that when designing this type of structures by the Subgrade Reaction Method the differences as compared with the results obtained by using specialized software can be substantial depending on the assumed load and support pattern. That is, the two methods for investigating anchored (supported) walls yield very different results with respect to their stress/strain state.

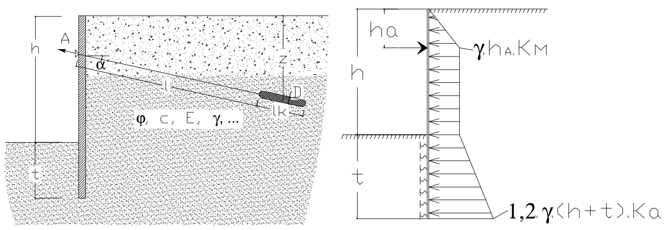

Fig.1.Anchored wall(a); Assumed load (b)

The article aims to search for such a method of determining the load acting on the wall and its support for which the results obtained by using a mass software package for statistical analysis are similar to both the experimental results and those obtained by using a specialized program of the PLAXIS type.

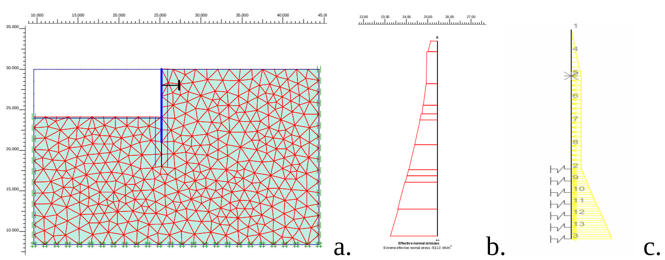

Fig.2. Numerical scheme according to PLAXIS (a), SAP (c) and active earth load according to PLAXIS (b)

It is assumed that in practice one-level anchored support is built for excavations of 4-10m in height ***h*** according to the engineering-geological and hydrogeological conditions. The driving depth ***t*** is assumed to vary within 0.25h - 1.00h (Fig.1). In this study we have assumed that the location of the anchor in height should be at a distance of 1/3h from an upper wall edge for all cases considered.

The analysis was carried out for a medium of homogeneous soil with characteristics varying within h=4-10m; t=0.25-1.00h; d=0.3-1.00m; φ=20-40°; c=0-20kPa; E=10000-30000kN/m^2; γ=10-20kN/m^3; ν=0.2-0.4; δ=0-0.67φ.

2 LOAD DETERMINATION

In the literature and technical standards there are various earth load patterns described in detail in (Totsev 2006). The presence of an anchor support at one level results in redistribution of load due to soil arching and occurrence of stress in the prestressed anchors. Some authors such as Tschebotarioff (1948, 1949), Rowe(1952) and Terzaghi (1955) suggest that the load should be considered linearly, varying in depth with a possible correction for the maximum value of the wall abutment level. Verdeyen, Roisin and Nuyens (1971), in turn, propose a correction (increase) of load in the anchor zone. In compliance with the investigation carried out for clarifying the earth load pattern, with the SAP results being closest to the PLAXIS ones for the same input parameters, the load distribution is assumed according to Fig.1,b.

The lateral pressure coefficient K_m here is the load coefficient. To determine K_m and prove the correctness of the proposed dependences, we solved in PLAXIS all possible combinations for walls with heights of h=4, 6, 8 and 10m, driving depth of t=0.25h, 0.50h, 0.75h and 1.00h and an angle of internal friction φ=20°, 30° and 40°. The results obtained serve as a point of departure for designing the same parameters with SAP. Varying with these parameters, we sought to equalize the results for the ultimate bending moments, ultimate Q-forces and anchor force A.

A similar solution by the two software packages and the type of results obtained are shown for one of the cases discussed (h=6m; t=0,50h=3m; E=10000kn/m^2; γ=20kN/m^3; φ=20°; c=0). Fig. 2 presents the numerical models according to SAP and PLAXIS as well as the active earth load obtained according to PLAXIS.

As can be seen from the figure, the size of the assumed load differs from that obtained by PLAXIS but, as we have already mentioned, our aim is to find an active load distribution for which the anchor force values and shear forces in the structure should be maximally close according to the two solutions. The diagrams obtained for M and Q are close not only as maximum dimensioning values but also in type. The value of the anchor force calculated by the two software programs is A=162.8*kN* (PLAXIS) and A=173.9*kN* (SAP), respectively. A similar approach was used in solving all other cases. The solutions obtained are the optimally close solutions found, varying with the parameters given. The weight falls on the closeness in the values of the anchor force and bending moments.

As a whole, the differences exceed 10% only in individual cases. We consider such a difference to be acceptable in terms of the simplification and unification of the approach proposed for applying SRM. We do not try to achieve absolute coincidence of results since that would lead to an exceptionally varied and complicated formulation. The aim is to find that "universal" scheme for determining the load and support for which the results are maximally close to the ones obtained by PLAXIS, which we assume to be reliable (with a deviation between 10-15%).

Km varies within a wide range and is affected by a number of parameters. For example, with increasing the angle of internal frictionφ, K_m decreases, and at φ=40° it is still twice as wide as that according to Rankin. It turns out that $K_m=f(\varphi)$ is not actually influenced by h. Therefore, we can draw a basic conclusion that K_M depends insignificantly on h and t/h. All dependences obtained and inferences made have been deduced for a medium having the following indices: E=10000kN/m^2, c=0, γ=20kN/m^3, φ=20-40°; for a 50cm thick wall with an unyielding anchor support. Subsequently, an analysis was conducted to determine the degree of influence of E_{soil}, c, ν, γ, EJ and C_A on the results for coefficient K_m. Many parallel calculations were performed for various possible combinations of E_{soil}, c, ν, γ, EJ and C_A varying within the range d=0.3-1.00m; c=0-20kPa; E=5000-15000kN/m^2; γ=10-20kN/m^3; ν=0.2-0.4 and C_A=10000-20000kN/m:

a. Effect of soil E-module

Comparison shows that the differences, with some exceptions, are within 5-10%. This practically means that if we assume the dependences obtained for E=10000 kN/m^2 to be reliable, then the error when working with other moduli would come up to 5% which we also consider acceptable. The results obtained enable us to use the dependences related to K_m for arbitrary values of the soil E-module.

b. Effect of cohesion (c).

The sequence of investigation thus presented was applied for cohesionless soils with zero cohesion. Correction of the load coefficient K_m is sought as a function of cohesion c. Such an approach contradicts the classical load patterns but the results obtained and the closeness of those obtained by PLAXIS give

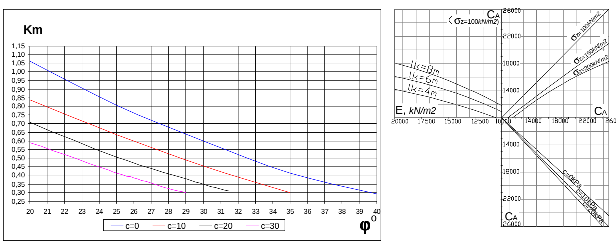

Fig.3. Variation in K_m as a function of φ and c (a). Determining the elastic constant of the anchor (b)

us good grounds to accept it. For a number of examples solutions by SAP and PLAXIS were carried out by searching for the value K_m for which the results are maximally close. As a generalization of the calculations performed we obtained Fig.3,a where, besides the effect of the angle of internal friction φ the effect of cohesion **c** on the lateral pressure coefficient K_m is taken into account.

c. Effect of wall stiffness (EJ).

When analyzing the effect of the wall stiffness EJ on the forces acting on the structure using PLAXIS it turns out that for the assumed modeling of the anchor as an unyielding support, a variation in the wall thickness within 0.40-1.00m at a constant E-module results in a minimum, proportional increase in the force acting on the anchor (up to 2.5% difference for the two boundary cases). The increase in the maximum moments and maximum Q-forces in the wall is also proportional to the increase in stiffness but here the differences are considerable (over 30%). When analyzing the behavior of the same wall using the SAP program by the spring-reaction method, then the dependences described above are confirmed, preserving the tendency of increasing the stiffness with increasing the force acting on the anchor, ultimate moments and ultimate Q-forces. Therefore, as was the case with determining K_m, it is not necessary to introduce a correction which accounts for the variation in wall stiffness.

d. Effect of yieldability of anchor (stiffness C_A).

To estimate the error and analyze the behaviour of the structure when modeling the anchor as a yielding support (spring) with a specified stiffness, two cases were considered. The anchor elastic constant was assumed to be C_A=10000kN/m and C_A=20000kN/m, respectively. The results obtained show clearly that with respect to the shear and anchor forces the solution for an unyielding support gives values close to those for a yielding one. This is not the case when determining the deformations (displacement) of the wall. In a more accurate solution with regard to the forces and mainly the displacements, the value of the anchor elastic constant C_A should be determined accurately depending on the particular terrain conditions. In section III a possible approach to finding C_A is presented.

e. Effect of the volume weight of soil

The results presented so far have been obtained for volume weight of soil γ=20 kN/m^3. We investigated how the variation in the volume weight would affect the final results. The volume weight variation within 10-20kN/m^3 is discussed. The variation in volume weight leads to a correction in the load by both SAP and PLAXIS. The results show that the dependences derived are not affected by γ and the empirical and graphic relationships proposed are valid for any volume weight.

3 SUPPORT

By analyzing various schemes for the value and distribution of the spring stiffness in depth, it turned out that optimally close results were obtained when assuming a linearly increasing stiffening of the proportionality coefficient in the form of a reduced triangle in depth (Fig. 4). The linear distribution in depth is corrected to trapezoid distribution by decreasing the spring rigidities for the first and last element from t. A final distribution is obtained as shown in Fig. 4. Apart from the soil module K_z, it also depends on the length of the (finite) soil-substituting elements Δ into which the wall is divided. A dependence was found for K_z and for the dimensionless coefficient β_K=f(t/h). The analysis of the numerical results obtained enables us to propose the relationship between the bed stiffness and soil elastic module in the form: $K_{z1,\max} = f(\beta_1, E, \Delta)$

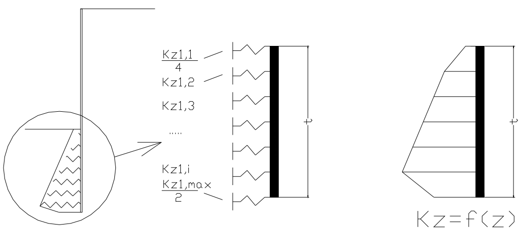

Fig.4. Distribution of K_{z1} in depth *Dependence A-δ*

The maximum value of the proportionality coefficient is suggested to be determined by the formula $K_{z1,\max} = \beta_1 \cdot E \cdot \left(\frac{\Delta}{\Delta + 2}\right)$ (1)

where E - strain module; β_1 –correction coefficient; Δ - length of the finite element according to dimension t. After determining the maximum value of K_z by formula (1), it is easy to determine the rigidities of all springs. The difference Δ_κ by which each subsequent spring rigidity decreases is: $\Delta_\kappa = K_{z1,max}/n$ (2), where n is the number of springs, i.e. $K_i = K_{z1,max} - \Delta_\kappa$ (3), $K_{i-1} = K_i - \Delta_\kappa$ etc. The value of the last spring (in the wall abutment) is assumed to be equal to 50% of the design value ($K_{z1,max}$) because the substituting soil width is ½Δ. Similarly, the spring at foundation pit bottom level is assumed to be equal to 25% of the value obtained assuming an additional decrease by 50% which accounts for the soil discompaction. Coefficient β_1 is equal to 4.2 and does not depend on h, t/h, φ and E.

4 DETERMINATION OF THE ANCHOR ELASTIC CONSTANT C_A

When designing anchored support structures, a problem arises how to model the anchor in the adopted numerical scheme. In engineering practice there are different variants for modeling the anchor as: 1) an unyielding support; 2) a yielding spring support of definite stiffness; 3) an active force directed toward the soil mass; 4) a rod of definite rigidity and length. Normally, even with priestesses anchors there is certain wall displacement in the anchorage area. Therefore, we assume that the anchor modeling in the form of yielding spring of definite stiffness (rigidity) is closest to the actual operation of the structure. As was found by the analysis carried out, the value of the anchor elastic constant $\boldsymbol{C_A}$ has a considerable effect on the results related to the displacements in the wall. Based on the definition of $\boldsymbol{C_A}$**,** namely, 'ratio of force to displacement', and since from a practical point of view displacements larger than 5cm for the wall anchorage area are admissible only in very rare cases, when determining the anchor elastic stiffness $\boldsymbol{C_A}$ we will assume that the values obtained refer to displacements within 0-5cm. As shows the analysis, the variation of the function in this section is very close to linear, i.e. $\boldsymbol{C_A}$ is a constant. Therefore, $\boldsymbol{C_A}$ is the relationship of the force in the anchor for which we have a displacement of 5cm toward the value of that displacement. To evaluate and estimate the effect of each parameter on the value of the anchor elastic stiffness, a set of problems were solved with parameters varying within σ_z= *100-200kN/m*^{2}*;* φ*=20-40°; c=0-20kPa;* l_k*=4-8m; E=10000-20000 kN/m*^{2}. As a result of the multiple solutions performed, we obtained Fig. 3,b, (Totsev 2006) from which, depending on the value of the parameters affecting C_A, we can determine the stiffness of the anchored support for a particular case.

5 DETERMINATION OF THE MINIMUM DRIVING DEPTH

Another basic problem in designing retaining walls is associated with determining the required minimum driving depth t. The main idea is to meet the condition that the soil under the foundation pit bottom should not lose stability resulting from applied load. In searching for a general approach to determining the necessary driving depth under varied soil conditions for different types of retaining wall at different pit height and cross-section, it is generally assumed that we should provisionally "shear" the wall at "pit bottom level". Subsequently, during the analysis we consider only the driven portion of the structure by substituting the effect of the supported soil mass by introducing an M and Q-force at the top of the wall section analyzed and the respective distributed load q according to Fig. 5. Hence, in determining t_{min} we are not interested in the wall type. The presence of supports as well as the type and distribution of the active load are selected at the discretion of the designer. Their effect on the determination of t_{min} is reflected by the variation in values and direction of M and Q.

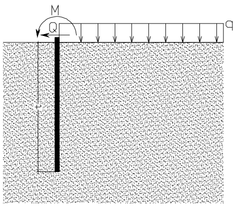

Fig.5 A model for t_{min}

Table.1 Limits of variation in input parameters

Parameters	Range	Parameters	Range
φ [°]	20--40	γ [kN/m^3]	10--20
c [kPa]	0--20	M [kN.m]	-150/+150
E [MN/m^2]	10--30	Q [kN]	0--150
ν	0,2--0,4	q [kN/m']	40--100
δ [°]	0--0,67φ	dwall [cm]	30--80

By using PLAXIS accurate solutions were obtained for the driven portion of the wall for different values of M, Q, q, d_{wall}, γ, φ, c, ν, δ and E_{soil}. It is assumed that the parameters vary within the limits shown in Table 1. A preliminary analysis has shown that the effect of certain parameters is essential (M, Q, γ, φ, c) whereas others have a weak effect regarding practical accuracy (q, d_{wall}, ν, δ и E_{soil}). It is a problem to assess the effect of each parameter in terms of the non-linearity in changing t_{min} with changing the respective parameter. The overall solution to the problem requires quantitative assessment of the group effect of all indicated parameters. Finally, we decided to study in detail the parameters (M, Q, γ, φ, c) whose variation affects considerably the value of t_{min}. The effect of the remaining parameters (q, d_{wall}, ν, δ and E_{soil}) is presented by a generalized coefficient. The main aim in this section is to find graphic or analytical dependences for direct determination of the required minimum driving depth. All possible combinations for the "input" parameters described have been solved which are assumed to vary within limits close the actually possible limits. Some combinations are purely theoretical and could not be obtained in practice but our exhaustive analysis requires their consideration.

The results obtained and the inferences proceeding from the numerical analysis conducted have been summarized graphically in Fig. 6. The basic conception is that by finding the respective M, Q, φ, c and γ for the particular problem, we should be able directly, by a series of steps in the abovementioned nomogram, to determine the necessary minimum driving depth t_{min} for which the structural stability conditions are fulfilled. The complexity of the problem stemming from the great variety of parameters influencing the final results makes it impossible to determine directly and precisely the required minimum driving depth. The boundary areas of t_{min} variation depending on one or several parameters have been involved in constructing the nomogram. When working with that nomogram, it is necessary to proceed from the particular conditions of the problem. For concrete input parameters t_{min} is obtained within certain limits. Its accurate adoption is a matter of designer's assessment and desicion. The graphic approach proposed for determining t_{min} is incomplete since it does not take into account the behavior resulting from a change in q, d_{wall}, ν, δ and E_{soil}. When determining the required minimum wall driving depth, we suggest that their effect should be estimated by using a generalized coefficient k_t. This coefficient is obtained as a sum of five independent coefficients kq, kd, kν, kδ, k_E. The value of each one is obtained as a result of the numerical analysis conducted and can be found by linear interpolation for the respective parameters from Table 2.

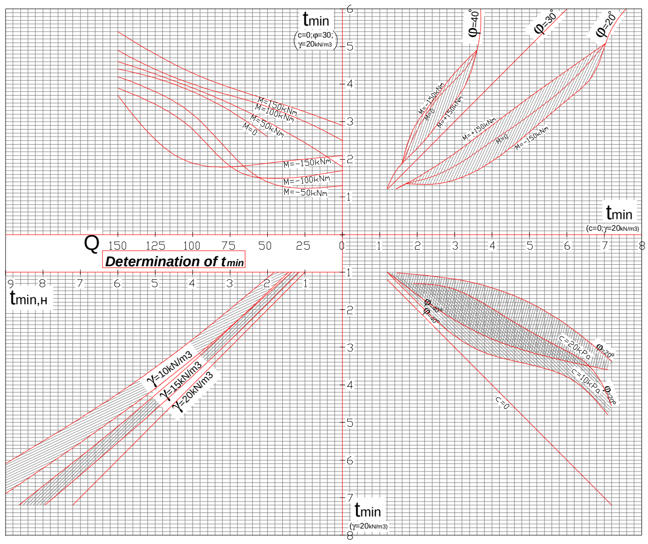

Fig.6 Direct determination of the minimum driving depth t_{min} for different values of M, Q φ, c and γ

Table 2. Determination of coefficients kq, kd, kν, kδ, k_E.

	d=30cm	d=80cm		δ=0,333φ	δ=0,667φ
kd	1	1,05	Kδ	1,15-1,20	1
	E=10MN/m^3	E=30MN/m^3		q=40kN/m'	q=100kN/m'
kE	1	0,94	kq	0,98	1,04
	ν=0,2	ν=0,4			
Kν	1	0,97			

For t_{min} we obtain: $t_{min} = t_{min,H} \,.\, k_t$ (4), where $k_t = kq.kd.k\nu.k\delta.k_E$ (5) ; $t_{min,H}$ - taken from Fig.6.

6 MULTILAYERED MEDIUM

A disadvantage of the computational scheme presented here and, in particular, in the part related to load determination is the assumption of a one-layer medium. From a practical geological point of view the foundation pits are characterized by an exceptional variety of soil layers. For example, very often when strengthening a pit in the Sofia area, two, three or more qualitatively different geological layers open up in depth on the construction site. This variety of layers itself leads to variation in the failure pattern thus questioning the accuracy in unifying the load pattern and reducing the solution to determining an averaged coefficient of both active (K_a) and lateral (K_m) pressure. Such an approach would be possible if the soils were similar in type and value of parameters. In the cases when this condition is not fulfilled, the variety of layers should be taken into account in determining the load coefficient K_m. For each layer we operate with a load determined for the particular layer under concrete soil parameters.

$$P_i=\gamma_i.h_a.K_{m,i} \qquad (6),$$

As regards the maximum load value in the wall abutment, the calculations have shown that the adoption of an averaged load coefficient K_a and averaged γ value does not result in considerable changes in the final results and yields force values very close to those calculated by PLAXIS. This is the reason why we recommend the formula:

$$P_{пета}=1,2.\gamma_{cp}.(h+t).K_{a,cp} \qquad (7),$$

where $K_{a,cp}=\dfrac{K_{a,1}.h_1+K_{a,2}.h_2+K_{a,3}.h_3+....K_{a,i}.h_i}{h+t}$ (8)

$$\gamma_{cp}=\frac{\gamma_1.h_1+\gamma_2.h_2+\gamma_3.h_3+....\gamma_i.h_i}{h+t} \qquad (9).$$

The load distribution is shown in Fig.7.

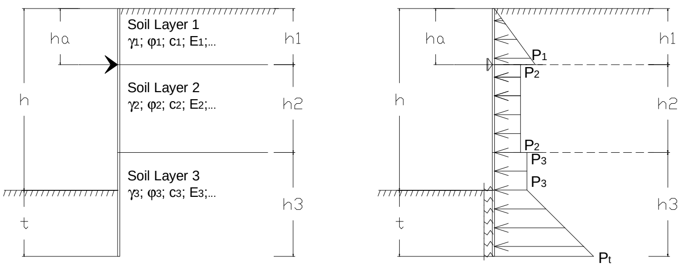

Fig.7. Solution for a multilayered medium

7 CONCLUSION

A number of dependences have been analyzed which relate to the effect of soil and geometric parameters on the stress/strain state of anchored (supported) retaining walls. The results obtained enable us to propose a transition from/to solutions realized by the specialized software package PLAXIS and the "non-specialized" use of SAP2000-type software programs. The procedure proposed is unified and provides adequate accuracy in terms of practical applications. It was recommend a type of load distribution on the wall, distribution of the stiffness of the elastic supports in depth beneath the foundation pit bottom as well as yieldability (by spring support stiffness) for an injection anchor with a clearly expressed base.

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