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Comparison of a gravimetric geoid model with data from GPS/levelling for the territory of South-western Bulgaria

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Abstract. *The present paper examines the accuracy of a local model of quasigeoid QBG01 for Bulgaria, as well as of EGG97 and EGM96 models for the territory of Bulgaria. The models have been compared by means of points in which highly precise GPS and levelling measurements have been performed, their number being totally 165. The working domain is restricted within the boundaries of 41° to 44.5° northern latitude and 22° to 28.5° eastern longitude. Differences between the models and the heights of the quasigeoid from GPS/levelling are within the range from ±1.228 m to ± 0.413 m. Four-parametric transformation was applied for their minimization and the accuracy of the models was respectively increased within the boundaries from ±0.243 m to ± 0.416 m. A restricted region in South-western Bulgaria (latitude of 42.05°–42.55° and longitude of 22.87°-23.7°) was investigated, for which higher data density was available. The mean square error of the deviations between GPS/levelling and QBG01 prior to transformation was ± 1.148 m, and after that it was reduced to ± 0.154 m. Conclusion may be drawn that both models – QBG01 and EGG97, are well representative and if sufficient GPS/levelling data are available, they may be used for interpolation of an erellipsoidal heights to normal ones with accuracy within the order of ±0.10 m and less.*

Introduction

Gravimetric and GPS/levelling methods are the main two modern approaches to determine the geoidal surface. Each of them possesses its own advantages and disadvantages.

The theory of the gravimetric method is known for a long time ago but its great practical importance has been found during the last decades with the possibility of obtaining local models using the remove-restore technique. In this technique gravimetric and topographic data for the studied region and a global geopotential model of the geoid are used. Most of the local geoid models of national importance have been obtained by means of this method. Accuracy of the gravimetric model depends basically on accuracy of the initial data and on their homogeneity and data distribution density. The theoretical models used for calculations and the applied numerical techniques also have impact in the achieved accuracy.

GPS/levelling method is relatively new one and acquired great practical importance with the possibility of obtaining ellipsoidal heights with sufficient accuracy. It is an attractive method because of the clear theoretical approach to its application. Its accuracy depends on the accuracy of determining two types of heights – ellipsoidal (from GPS) and levelling one. In general case, the insufficient density of the measured points is a disadvantage of the method.

Combination of the two methods represents an optimal variant for obtaining a model of the geoid. The attaining of the model by means of two independent methods provides the possibility of making an accuracy assessment for the developed gravimetric model by means of points with performed GPS/levelling. Combination of both methods eliminates their shortcomings –the model obtained by GPS/levelling is used to refer the gravimetric model to a unified reference and height system for the studied region. After presenting a combination of the two models interpolation of geoidal heights becomes possible for points with an available GPS positions. The latter has a very important practical application – derivation of the transformation model makes possible the direct transformation from ellipsoidal to normal height.

Accuracy assessment of three gravimetric geoid models for the territory of Bulgaria has been made in the present paper – 1) local model of quasigeoid QBG01 (Quasigeoid BulGaria 2001) for the territory of Bulgaria, 2) regional model of quasigeoid EGG97 (European Gravimetric Geoid 1997) for Europe and 3) global EGM96 model of the geoid (Earth Gravity Model 1996). The three models are successfully compared in points for which highly accurate GPS and levelling data are available. The working area is defined with respect to the QBG01 model and it is within the boundaries of geodetic latitude of 41° – 44.5° and longitude from 22° to 28.5°. The number of the points with data from GPS/levelling located on the territory of Bulgaria are 165.

Initial data

Gravimetric model QBG01

Fig.1. Gravimetric model of the quasigeoid QBG01, section of horizontals 0.5 m

The derived gravimetric model of the quasigeoid for Bulgaria QBG01 was calculated in 2001 (Peneva, 2001). The working area is within the boundaries of 41°– 44.5° northern latitude and 22°– 28.5° eastern longitude. The developed model is a local one – it reflects the shortwave variations in the gravitational field. Its calculation is realized using available gravimetric data with high density from surface measurements for the territory of the country. The remove-restore technique was applied with the global geopotential EGM96 model as a reference surface. The global topographic model DTED (Digital Terrain Elevation Data) Level 0, created by NIMA (National Imaging and Mapping Agency) with density of 30″ x 30″ was used, too. Gravimetric data were obtained from maps of the Bouguer anomalies in scale 1:200 000 covering the territory of the country with a section of 2 mGal, and a digital grid model with density of 1′ x 1′ was composed on their basis. Gravimetric data processing and quasigeoid model computation were realized using programmes of the GRAVSOFT software package (Tscherning, 1992). The height anomalies were calculated using the gradient solution method (Heiskanen and Moritz, 1967). The quasigeoidal model is shown in Figure 1 and the model statistics is presented in Table 1.

Gravimetric model EGG97

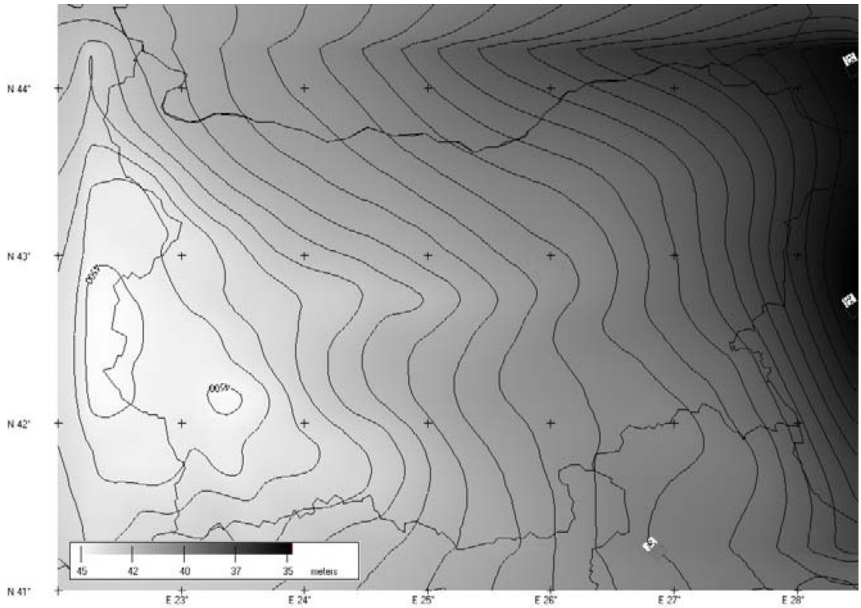

Fig.2. Gravimetric model of quasigeoid EGG97 for the studied territory, section of horizontals 0.5 m

The gravimetric model of quasigeoid EGG97 comprises the territory of Europe and was developed by a team of the Institute for Geodesy of the University of Hannover (Institut für Erdmessung IfE, University of Hannover, Germany) under the guidance of IAG Geoid Commission (Denker et al., 1997). The computational technique is based on application of the remove-restore technique and spectral analysis for data combination. The global geopotential model EGM96 was used for reflecting the longwave variations of the gravitational field. A total of 2.7 million gravimetric data were collected and 700 million topographic data were processed in the form of a digital model of the relief with density of 7.5″ x 7.5″ and residual gravimetric anomalies with density of 1′ x 1.5′ for the territory within 25°–77° northern latitude and 35°–67.4° eastern longitude. The density of the model is 1′ x 1.5′ for the region of West Europe and 10′ x 15′ for the eastern European countries. The territory of Bulgaria falls within the region with lower density of the model (10′ x 15′). The quasigeoid model EGG97 for Bulgaria is presented in Figure 2 and its basic characteristics are shown in Table 1.

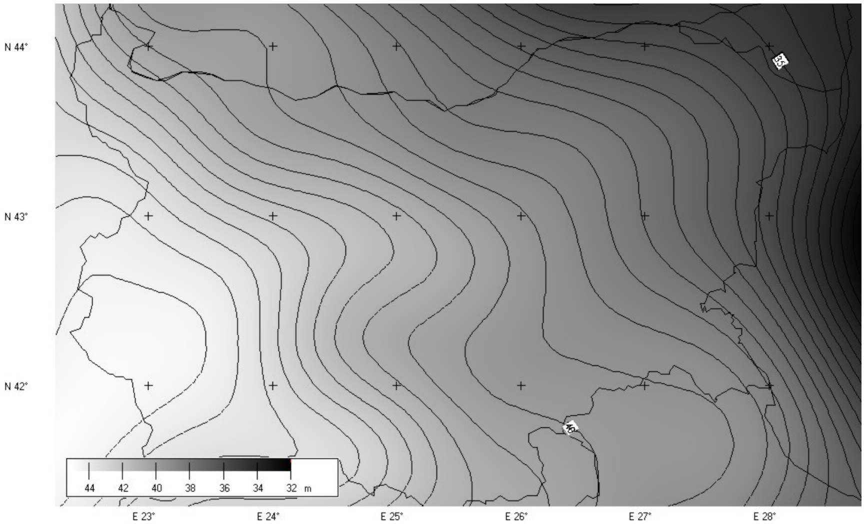

Fig.3. Global model of geoid EGM96 for the studied territory, section of horizontals 0.5 m

Global geoid model EGM96 was developed as a result of the joint work of NIMA and NASA/GSFC (National Aeronautics and Space Administration's Goddard Space Flight Centre) and at presents it is the most representative one. It is based on various sources of gravimetric data covering the whole Earth's surface, which have been collected and processed in the form of a regular network from free air anomalies (Kenyon, 1997, Lemoine et al., 1997). The model consists of a grid network of average free air anomalies with spacing of 30′ x 30′ for land territories and 1° x 1° for ocean areas. Height databases with grid density of 1′ and 5′ JGP95E (Joint Gravity Project 95) composed by NIMA and GSFC have been used for introducing the topographic reductions. Computations are based on the collocation method according to LSM (the Least Square Method), JGM-2/OSU91A geoid model (Joint Geopotential Model 2/ Ohio State University 91A) being used as the reference one for longwave effects. The geoid model EGM96 for the territory of Bulgaria is presented in Figure 3 and its basic characteristics are shown in Table 1.

GPS/levelling data

The number of points used is 165, precise GPS measurements and levelling being performed for these points. Unfortunately, the points are not regularly distributed throughout the territory of the country (Figure 4) and they have been obtained from different sources. The coordinates of GPS points are in the World Geodetic System 1984 (WGS84) coordinate system, the agreement between WGS84 and ITRF2000 (International Terrestrial System 2000) is not higher than 2 cm in a global scale. The points, where GPS measurements have been carried out are levelling benchmarks from the National Levelling Network of Bulgaria (in Bulgaria it was accepted to work with normal heights after 1950). The determination of normal heights is referred to the Baltic height system. We consider that the accuracy of the ellipsoidal heights from GPS does not exceed 2 cm. Accuracy of the normal heights is better than 1 cm. The statistics of the used GPS/levelling data is presented in Table 1.

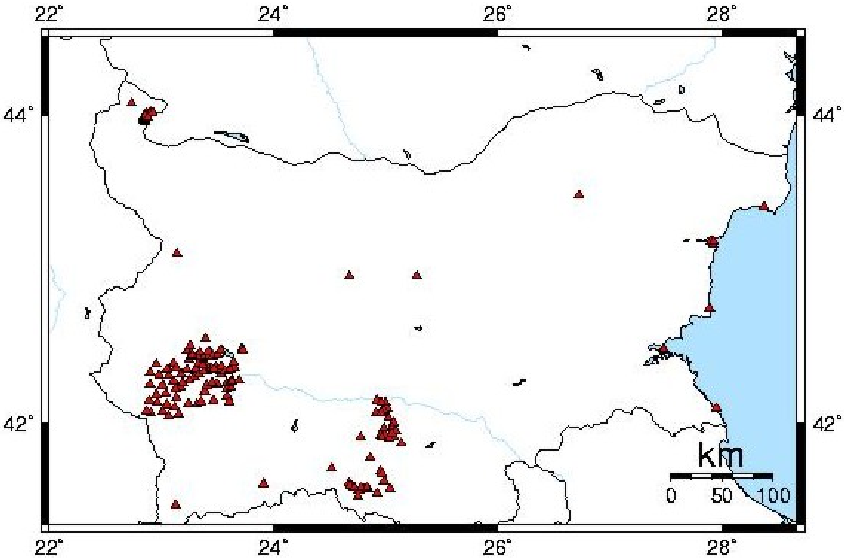

Fig.4. Distribution of the points with data from GPS/levelling

T a b l e 1. *Statistics of the used models and data for the studied territory, in [m].*

	max	min	mean	std
EGM96	45.01	33.40	40.11	± 2.85
EGG97	45.28	34.74	40.92	± 2.55
QBG01	45.26	33.02	40.11	± 2.65
H^N	2925.087	-0.111	794.767	± 580.070
H^{GPS}	2971.314	36.075	838.820	± 581.689

Comparison of the geoid model with GPS/levelling

The comparison became from the clear relationship between the three height types – normal height H^N, geodetic height H and height anomaly ζ:

(1) $$\zeta = H - H^N \ .$$

Normal height is obtained by means of precise levelling and the geodetic height may be obtained from GPS determinations. The height anomaly can be respectively obtained applying formula (1), when H and H^N are known, i.e. which represents in fact the GPS/levelling method. The height obtained in such a way is designated as ζ^{GPS}. GPS measurements of levelling benchmarks have to be available for its establishment. Quasigeoid surface may be obtained also by means of the gravimetric method, the height anomaly thus obtained being designated as ζ^{grav}. To determine the differences of height anomalies, it is necessary to interpolate values for ζ^{grav} for the points with ζ^{GPS} values. The comparison of height anomalies obtained by GPS/levelling ζ^{GPS} with anomalies yielded by gravimetric geoid models may be used for evaluation of their accuracy. The differences between both anomalies contain the errors from determining the three height types – normal, geodetic and quasigeoidal heights obtained according to the gravimetric method. The main influence is that of the systematic errors due to inhomogenity of the initial data which are in referring them to different reference systems. Differences from the two models may be presented by a four-parametric transformation model (Heiskaknen and Moritz, 1967, Kostakis and Sideris, 1999, Martensson, 2002):

$$\zeta_i^{GPS} - \zeta_i^{grav} = a_0 + a_1 \cos\varphi_i \cos\lambda_i + a_2 \cos\varphi_i \sin\lambda_i + a_3 \sin\varphi_i + v_i \quad (2)$$

A four parametric adjustment model is applied to obtain a_i coefficients, composing equations of type (2) for each point *i* of the data with geodetic coordinates (φ_i, λ_i). As a result, values of the transformation parameters a_0, a_1, a_2, a_3 and corrections v_i are obtained. The problem is that v_i values contain errors of various character – a combination of random errors of GPS and levelling determinations and calculation of the gravimetric geoidal height. The joint adjustment of the three height types yields also a correction surface for direct transformation from geodetic height obtained by GPS measurements to normal height, which is of great practical importance.

Results

The three gravimetric models of the geoid for the territory of Bulgaria – QBG01, EGG97 and EGM96, were compared with 165 points with high accurate levelling and GPS determinations. The boundaries of the studied area are between 41° – 44.5° northern latitude and 22° – 28.5° eastern longitude. Statistics of the obtained results is shown in Table 2 and Table 3. Table 2 presents differences before applying four-parametric transformation and Table 3 – after its completion.

As it is seen from Table 2 and Table 3, the best results are exhibited by the European quasigeoid EGG97 model – the mean square error of the differences before transformation is ± 0.413 m, and after it ± 0.243 m. The global EGM96 model is characterized by rather lower accuracy of ± 1.1431 m before and ± 0.416 m after transformation. The local quasigeoid model QBG01 for Bulgaria exhibits mean square error of ± 1.228 m before and ± 0.374 m after transformation.

T a b l e 2. *Statistics of differences between the geoid models and the heights from GPS/levelling prior to application of the four-parametric transformation, in [m].*

Geoid model/ Statistics	**max**	**min**	**mean**	**rms**
ζ^{GPS} - N^{EGM96}	2.342	0.000	1.022	± 1.143
ζ^{GPS} - ζ^{EGG97}	0.905	-0.647	0.325	± 0.413
ζ^{GPS} - ζ^{QBG01}	1.939	-0.326	1.132	± 1.228

T a b l e 3. *Statistics of differences between the geoid models and the heights from GPS/levelling after completion of the four-parametric transformation, in [m].*

Geoid model/ Statistics	**max**	**min**	**mean**	**rms**
$\zeta^{GPS} - N^{EGM96}$	1.329	-0.891	0.000	± 0.416
$\zeta^{GPS} - \zeta^{EGG97}$	0.572	-0.961	0.000	± 0.243
$\zeta^{GPS} - \zeta^{QBG01}$	0.745	-1.108	0.000	± 0.374

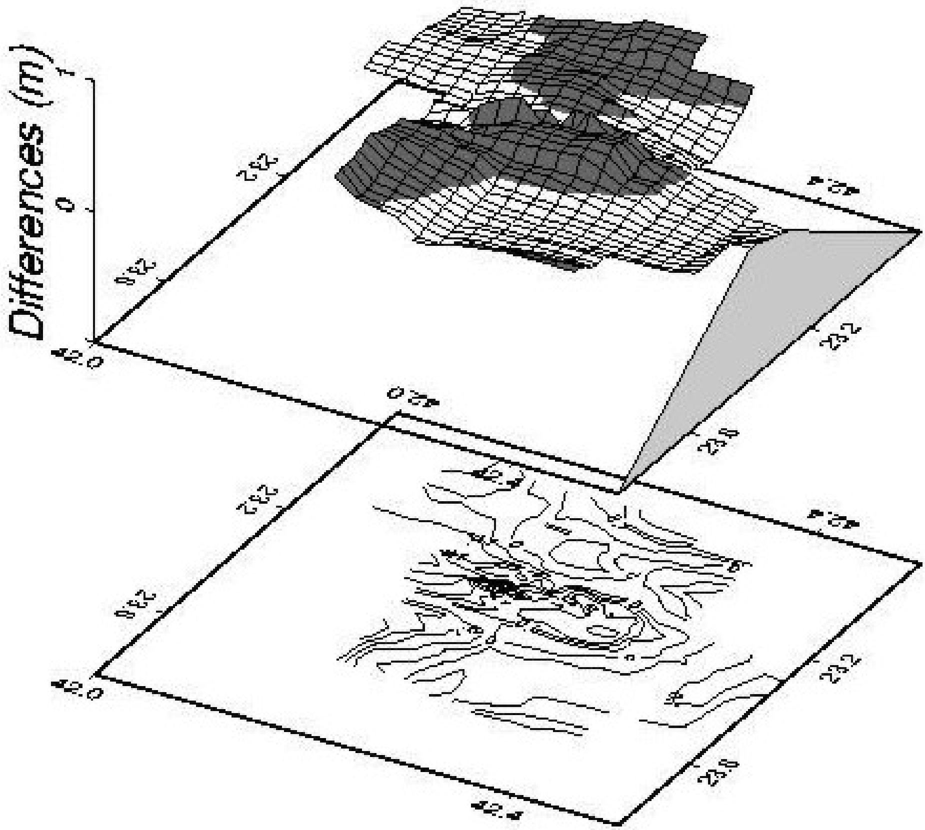

Fig.5. Model of differences between the QBG01 model and the heights from GPS/levelling for a territory restricted within φ 42.05°– 42.55° and λ 22.87°– 23.7° after applying the transformation, in [m]

The lower accuracy of the local quasigeoid model QBG01 is due mainly to inhomogenity and systematic errors in the initial data. The lack of gravimetric data for the neighbouring countries has also impact on the accuracy. After applying the four-parametric transformation the accuracy is significantly increased due to removal of the systematic errors during computation.

The irregular distribution of the points with GPS/levelling and their low number (Figure 1) represents a shortcoming of the performed comparison. Another investigation has been carried out for the region of South-western Bulgaria with higher data density and regular data distribution. The studied territory is within the range of 42.05° – 42.55° northern latitude and 22.87° – 23.7° eastern longitude. The number of points with GPS/levelling for this region is 85, the average density between the points is 8.5 km. The values from the comparison of these points with the gravimetric model QBG01 are given in Table 4. The mean square error of the differences is ± 1.148 m, and the accuracy is significantly improved after applying the transformation for the region – ± 0.154 m. The surface of the differences obtained after applying the transformation is presented in Figure 5. It is shown with isolines and grid model, the dark sections are the differences with positive values and the light ones are the differences with negative values.

The results show that when data with higher density are available, the considered geoid models may be significantly improved in decimetre level accuracy may be achieved in interpolation of height anomaly.

T a b l e 4. *Statistics of differences between the QBG01 model and the heights from GPS/levelling for a territory with φ 42.05 ° – 42.55 ° and λ 22.87 ° - 23.7 ° prior to and after applying the four-parametric transformation, in [m].*

Dupnica-Samokov	**max**	**min**	**mean**	**rms**
$\zeta^{GPS} - \zeta^{QBG01}$ ***(before)***	1.665	0.000	1.106	± 1.148
$\zeta^{GPS} - \zeta^{QBG01}$ ***(after)***	0.554	-0.256	0.000	± 0.154

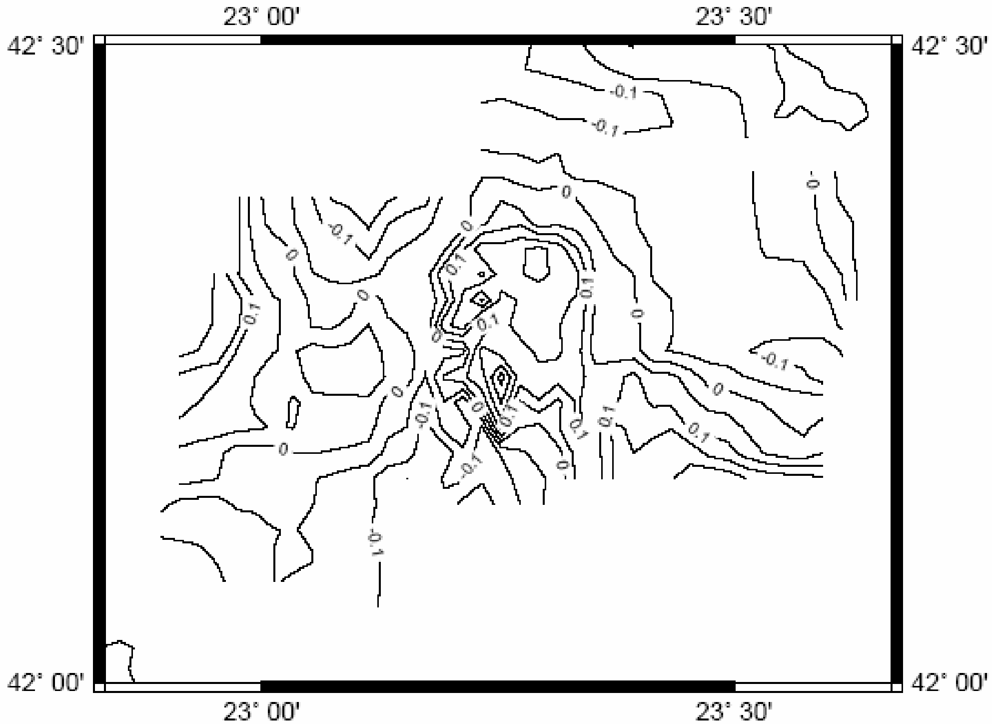

Fig.6. Isolines of the differences between the QBG01 model and the heights from GPS/levelling

Conclusions

The investigation of QBG01 and EGG97 gravimetric models shows that they have good representativeness. Accuracy of the European model of the quasigeoid EGG97 for Bulgaria is little higher than the local model QBG01. However, QBG01 model represents better the shortwave variations of the gravitational field, the density of this model is 1′ x 1′ compared to EGG97 model having density of 10′ x 15′ for the Bulgarian territory. The last investigation for a studied of the South-western Bulgaria region with significantly higher density of GPS/levelling points (Table 4) let us conclude that QBG01 and EGG97 models could be applied for transformation of ellipsoidal heights into normal ones with accuracy of the order of a decimetre for the whole territory of the country if enough GPS/levelling date are available.

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Сравнение на гравиметричен модел на геоида с данни от GPS/нивелация за територията на Югозападна България

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(Р е з ю м е)

В настоящата статия се изследва точността на локалния модел на квазигеоида за България QBG01, както и моделите EGG97 и EGM96 за територията на България. Моделите са сравнени с точки, в които е са извършени високоточна GPS и нивелачни измервания – 165 на брой. Работната област е ограничена в границите от 41° – 44.5° северна ширина и от 22° до 28.5° източна дължина. Разликите, които се получават между моделите и височините на квазигеоида от GPS/нивелация са в границите от ±1.228 m до ± 0.413 m. Приложена е четири-параметрична трансформация за минимизирането им, съответно точността на моделите се повишава в границите от ±0.243 m до ± 0.416 m. Направено е изследване за ограничен район в Югозападна България (42.05°–42.55° ширина и 22.87°-23.7° дължина), където се разполага с по-голяма гъстота на данните. Преди трансформацията средната квадратна грешка на отклоненията между GPS/нивелация и QBG01 е ± 1.148 m, а след трансформацията пада на ± 0.154 m. Може да се направи заключение, че двата модела QBG01 и EGG97 са достатъчно представителни и ако се разполага с достатъчна гъстота на данни от GPS/нивелация, могат да се ползват за интерполация на наделипсоидни височини в нормални с точност от порядъка на ± 0.10m и по-малка.